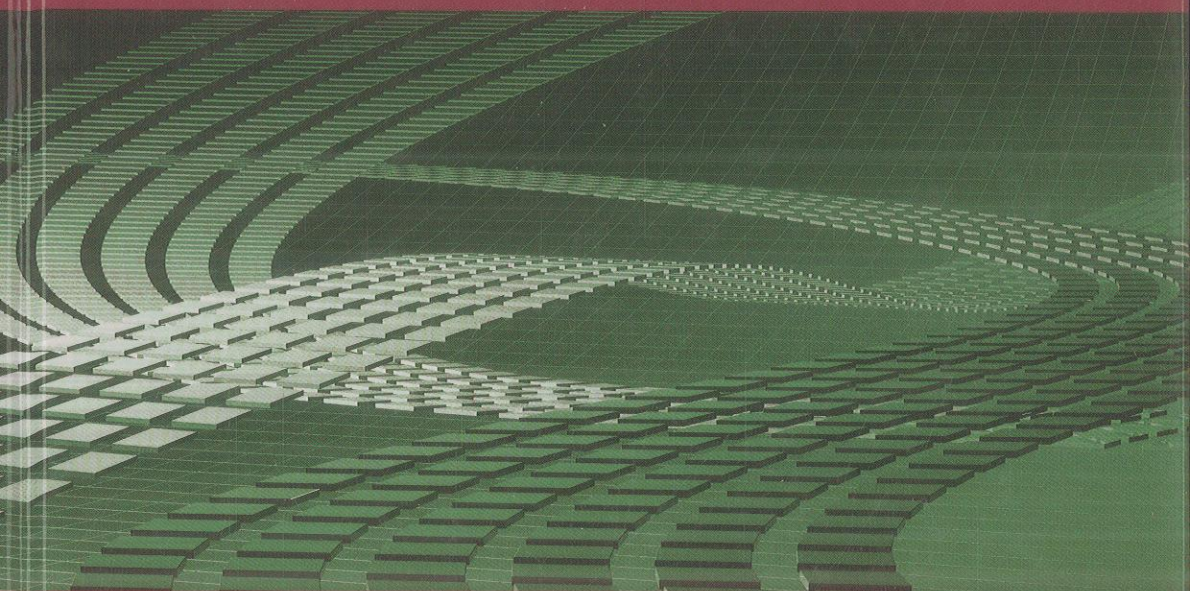


Mechanical Vibration and Shock Analysis



Random Vibration

Volume 3

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ISTE

 **WILEY**

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